Spectrum and strong couplings of heavy-light hybrids

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The spectrum of the 0^{++}, 0^{--}, 1^{-+}, and 1^{+-} heavy-light hybrids are calculated in HQET. The interpolated currents of the hybrids are chosen as $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a h_v(x)$, $g\bar{q}\gamma_\alpha\gamma_5 G^a_{\alpha\mu}T^a h_v(x)$, and $g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a h_v(x)$. Some strong couplings and decay widths of the heavy-light hybrids to $B(D)\pi$ are calculated by using QCD sum rules. The masses of 0^{++} hybrids with a gluon in TM(1^{--}) and TE(1^{+-}) modes are found to be similar, while the decay widths of them are found to be different. A two-point correlation function between the pion and vacuum is employed and the leading order of $1/M_Q$ expansion is kept in our calculation.

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I. INTRODUCTION

It has been almost twenty years since we began to search for the exotic hadrons such as the glueballs and hybrids. There are some special states which are regarded as candidates for hybrids; especially $\hat{\rho}(1400)$ and $\hat{\rho}(1600)$ have been studied widely, but no confirmation has been made so far. Recently, these two special states have aroused great interest again. The E852 Collaboration at BNL [1] has reported a $J^{pc}=1^{-+}$ isovector resonance $\hat{\rho}(1405)$ in the reaction $\pi^- p \to \eta\pi^0 n$. The mass and width of this state have been reported with $1370\pm16^{+50}_{-30}$ MeV and $385\pm40^{+65}_{-105}$ MeV, respectively. The Crystal Barrel Collaboration has also claimed to have found an evidence in $p\bar{p}$ annihilation which may be a resonance with a mass of $1400\pm20\pm20$ MeV and a width of $310\pm50^{+50}_{-30}$ MeV [1]. The confirmation of these states will provide some evidence for the existence of hybrids. At present, all the experiments specialize in the light quark hybrids, but it is necessary to extend the energy region to hybrids including the b or c quark, which is also possible in the B or $\tau-c$ factory.

Theoretically, the spectrum and decay width of hybrids have been calculated widely with many methods, such as the bag model [2] and flux-tube model [3], QCD sum rules [4], lattice [5] and other models [6]. However, there are few works on the spectrum and decay width of heavy-light hybrids [7]. Heavy quark effective theory (HQET) has led to much progress in the theoretical understanding of the properties of hadrons [8]; one may wonder whether the effective theory is suitable to deal with the heavy-light hybrids or not. Especially, the sum rules' calculation for the heavy-light hybrids in full QCD theory in Ref. [7] shows that the component of gluon gives a contribution of more than 1.0 GeV to the mass of hybrids, so the "light freedom" of hybrids seems too heavy to keep the $1/M_Q$ expansion available. The direct calculation of the spectrum of hybrids in HQET will give an answer to the question. Our results show that the spectrum of heavy-light hybrids including the b or c quark are close to those calculated in full QCD theory and it is suitable to deal with these hybrids within HQET. Besides, compared with the calculation for $b\bar{b}g$ and $c\bar{c}g$ hybrids in full theory, the calculation for the spectrum and decay widths of the heavy-light hybrids in HQET is much easier.

It is interesting for the experimentalists to search the exotic 1^{-+} heavy-light hybrids, so theoretical determination of the property of these states is necessary and urgent. In full QCD theory, the estimation in Ref. [7] showed that the sum rule for the mass of 1^{-+} heavy-light hybrids had no platform at all. The masses of 1^{-+} hybrids were given under the main assumption that the contribution of the gluon condensate is less than 20% of the bare loop. In HQET, the ambiguous situation has been improved greatly. The surface of the Λ versus Borel variable τ varies slowly in a large region, which gives a good platform. Then the masses of the hybrids are determined reasonably.

According to the MIT bag model [9], the hybrids with the same J^{pc} have different internal interactions between the partons which indicates that they are different states, that is to say, the gluon in hybrid can be in different mode [TM(1^{--}) or TE(1^{+-})]. In order to predict the properties of them, we should choose suitable generating currents corresponding to these states for the calculation. In the case of light quark hybrids [4], the different 0^{++} states with gluon in different modes were found to have different masses. In heavy-light hybrids' case, the calculation shows that the mass splitting of the different 0^{++} heavy-light hybrids with gluon in different modes is not large in the $M_Q\to\infty$ limit. The mass of 0^{++} hybrids from two different currents, $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a h_v(x)$ with gluon in TM(1^{--}) mode and $g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a h_v(x)$ with gluon in TE(1^{+-}) mode, is found similar.

Though the masses of these two different 0^{++} hybrids are

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similar, the decay widths for them to $B(D)\pi$ final states are found different: the decay width of hybrid from current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a h_v(x)$ is about 86 MeV or 16 MeV corresponding to $B\pi$ or $D\pi$ final states, respectively, while the one from current $g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a h_v(x)$ is only 11 MeV or 2.6 MeV.

The decay widths for the 0^{++} and 1^{-+} hybrids decaying to $B(D)\pi$ final states have been calculated in Ref. [10], where the decay constants of the hybrids were obtained from the formulas in full theory [7]. Both the strong couplings and the decay constants are calculated in HQET in this paper, and the strong coupling of the 0^{++} hybrid obtained here is much larger than that in Ref. [10]. The large difference is from the values of the decay constant calculated in two different ways.

In the calculation of the strong couplings, a two-point correlation function between the pion and vacuum is taken to avoid the ambiguity resulting from the double Borel transformation for the ordinary three-point correlation function between the vacuum and the infrared problem in the soft pion limit. For convenience, the calculation is kept in the leading order of $1/M_Q$ expansion.

The paper is organized as follows. The analytic formalism of HQET sum rules for the spectrum of hybrids is given in Sec. II. In Sec. III, we give the numerical results of the spectrum and decay constants of hybrids; the comparison of the spectrum with that in full QCD theory is given also. In Sec. IV, with the help of the two-point correlation function between the pion and vacuum, the analytic formalism of HQET sum rules for the strong couplings of hybrids is derived and the numerical results of some decay widths were obtained. In the last section, we give the conclusion and discussion.

II. HQET SUM RULES FOR THE SPECTRUM OF THE HEAVY-LIGHT HYBRID MESONS

As we know, the bag model [2] is a nonrelativistic model, in which the gluon in hybrid can be in a different mode [TM(1^{--}) or TE(1^{+-}) mode]. While the QCD sum rule is a relativistic one, there is no direct relation between them. Besides, there is no definite J^{pc} for the hybrid interpolating currents. However, it is possible to study these different hybrids with gluon in different modes by using QCD sum rules; the key is to choose suitable generating currents. From the analysis in Ref. [4] (Huang *et al.*), the mass of hybrid from current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x)$ is 1.0 GeV larger than that from current $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^a q(x)$. Compared with the results in the bag model, these two different states were regarded as two hybrids with gluon in TM and TE model, respectively. For the 0^{++} hybrid from current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x)$, the combination of J^{pc} for the quark pair is 1^{--} and the gluon is in TM(1^{--}) mode. For the 0^{++} hybrid from current $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^a q(x)$, the combination of J^{pc} for the quark pair is 1^{+-} while the gluon is in TE(1^{+-}) mode.

From the analysis above, in order to calculate the spectrum of the 0^{++} and 1^{-+} heavy-light hybrids with the gluon in TM(1^{--}) and TE(1^{+-}) mode, respectively, the interpolated current in HQET is chosen as

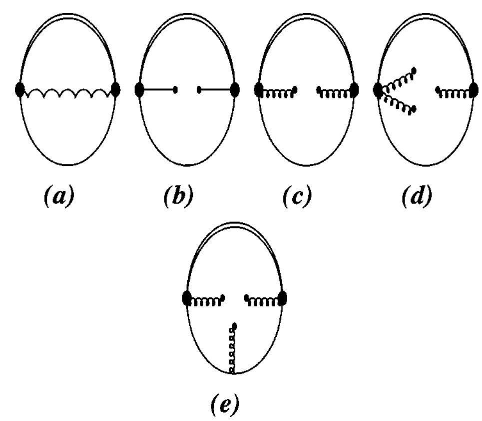

FIG. 1. Feynman diagrams contributing to the correlation function in HQET.

$$j_\mu(x)=g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a h_v(x), \tag{1}$$

where $q(x)$ is the light quark field and $h_v(x)$ is the heavy quark effective field; v is the velocity of the heavy quark.

Then, we construct the correlation function as

$$\Pi_{\mu\nu}(\omega)=i\int d^4x e^{iqx}\langle 0|T\{j_\mu(x),j^+_\nu(0)\}|0\rangle$$
$$=(v_\mu v_\nu-g_{\mu\nu})\Pi_v(\omega)+v_\mu v_\nu \Pi_s(\omega), \tag{2}$$

where

$$\omega=2q\cdot v. \tag{3}$$

Since the free heavy quark propagator in HQET is $\int_0^\infty d\tau\delta(x-v\tau)[(1+\not{v})/2]$ and the interaction of the heavy quark with the gluon field A_μ in the leading order of $1/M_Q$ expansion is $g\bar{h}v\cdot Ah$. Then under the fixed-point gauge $x_\mu A_\mu=0$ (which will be used throughout this paper), the full propagator of the heavy quark $\langle 0|T(h(x)\bar{h}(0)|0\rangle$ in the leading order of $1/M_Q$ expansion is the same as the free one. Then the freedom of the heavy quark can be extracted out of the matrix element as a delta function, which facilitates the calculation.

In the operator product expansion, we keep the perturbative term, two gluon condensate term, three gluon condensate term, and two quark condensate term. The contribution of mixing condensate and higher dimension operators are negligible for their smallness. The Feynman diagrams are shown in Fig. 1, where the double line represents the propagator of the heavy quark. After twice suitable Borel transformation, we obtain the $\mathrm{Im}\,\Pi_s(\omega)$ and $\mathrm{Im}\,\Pi_v(\omega)$ corresponding to the scalar and vector states' contribution to the sum rules, respectively,

$$\mathrm{Im}\,\Pi_s(\omega)=\frac{\alpha_s}{960\pi^2}\omega^6+\frac{\alpha_s}{160\pi^2}m\omega^5-\frac{1}{16}\langle\alpha_s G^2\rangle\omega^2-\frac{m}{8}\langle\alpha_s G^2\rangle\omega-\frac{\alpha_s}{6}\langle\bar{q}q\rangle\omega^3+\frac{\alpha_s}{4}m\langle\bar{q}q\rangle\omega^2-\frac{\alpha_s}{16}\langle gG^3\rangle,$$

$$\mathrm{Im}\,\Pi_v(\omega)=\frac{\alpha_s}{960\pi^2}\omega^6+\frac{\alpha_s}{480\pi^2}m\omega^5+\frac{1}{48}\langle\alpha_s G^2\rangle\omega^2+\frac{m}{8}\langle\alpha_s G^2\rangle\omega-\frac{\alpha_s}{18}\langle\bar{q}q\rangle\omega^3+\frac{\alpha_s}{4}m\langle\bar{q}q\rangle\omega^2-\frac{\alpha_s}{48}\langle gG^3\rangle, \tag{4}$$

where the light quark mass corrections are considered also in these formulas. From the numerical estimation in Sec. III, the nonperturbative contribution to the sum rules is only 10–20 %, so the truncation of the operator product expansion (OPE) is a good one.

As for the 0^{--} and 1^{+-} hybrids, the current was chosen as

$$j_{5\mu}(x)=g\bar{q}\gamma_\alpha\gamma_5 G^a_{\alpha\mu}T^a h_v(x). \tag{5}$$

The correlation function expanded is similar to the vector current case except for the opposite sign for the contribution of the quark condensates terms to the sum rules, and their spectrum will be determined in a similar way.

For the 0^{++} hybrid with the gluon in TE(1^{+-}) mode, the following current should be used to predict the mass:

$$j(x)=g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a h_v(x). \tag{6}$$

The $\mathrm{Im}\,\Pi(\omega)$ for this current has been carried out as

$$\mathrm{Im}\,\Pi(\omega)=\frac{\alpha_s}{120\pi^2}\omega^6-m\langle\alpha G^2\rangle\omega+2\alpha_s m\langle\bar{q}q\rangle\omega^2-\frac{\alpha}{16}\langle gG^3\rangle. \tag{7}$$

In the chiral limit, the contribution of two gluon and two quark condensates to the sum rules vanish.

On the phenomenal side, the decay constants of hybrids, $F_{H^\pm}$, are defined as

$$\langle 0|j_\mu|H(0^+)\rangle=F_{H^+}m_{H^+}^{1/2}v_\mu,$$
$$\langle 0|j_\mu|H(1^-)\rangle=F_{H^-}m_{H^-}^{1/2}\epsilon_\mu, \tag{8}$$
$$\langle 0|j|H'(0^+)\rangle=F'_{H^+}m'^{1/2}_{H^+},$$

where m_H represents the mass of hybrid, ϵ_μ is the polarization vector of the $H(1^-)$, and the two different 0^{++} hybrids with gluon in TM(1^{--}) and TE(1^{+-}) mode are labeled as $H(0^{++})$ and $H'(0^{++})$, respectively. So the correlation functions read

$$\Pi_s(\omega)=-\frac{F^2_{H^+}}{(2\Lambda-\omega)}+\int_{\omega_c}^{\infty}d\omega'\frac{\mathrm{Im}\,\Pi_s(\omega')}{\omega'-\omega},$$
$$\Pi_v(\omega)=-\frac{F^2_{H^+}}{(2\Lambda-\omega)}+\int_{\omega_c}^{\infty}d\omega'\frac{\mathrm{Im}\,\Pi_v(\omega')}{\omega'-\omega}, \tag{9}$$
$$\Pi(\omega)=-\frac{F'^2_{H^+}}{(2\Lambda-\omega)}+\int_{\omega_c}^{\infty}d\omega'\frac{\mathrm{Im}\,\Pi(\omega')}{\omega'-\omega}.$$

where the first term of the right side is the dominant pole term resulting from the lowest lying resonance contribution and the second term represents the contribution of the continuum state and higher resonances, ω_c is the continuum threshold.

Making use of the dispersion relations for the correlation functions to equate the quark and hadron sides, we obtain

$$\frac{F^2_{H^+}}{(2\Lambda-\omega)}=-\frac{1}{\pi}\int_0^{\omega_c}d\omega'\frac{\mathrm{Im}\,\Pi_s(\omega')}{\omega'-\omega},$$
$$\frac{F^2_{H^-}}{(2\Lambda-\omega)}=-\frac{1}{\pi}\int_0^{\omega_c}d\omega'\frac{\mathrm{Im}\,\Pi_v(\omega')}{\omega'-\omega}, \tag{10}$$
$$\frac{F'^2_{H^+}}{(2\Lambda-\omega)}=-\frac{1}{\pi}\int_0^{\omega_c}d\omega'\frac{\mathrm{Im}\,\Pi(\omega')}{\omega'-\omega}.$$

After the Borel transformation [11], they are turned into

$$F^2_{H^\pm}e^{-2\Lambda/T}=-\frac{1}{\pi}\int_0^{\omega_c}d\omega'\,\mathrm{Im}\,\Pi(\omega')e^{-\omega'/T}, \tag{11}$$

where T is the Borel transformation variable. Therefore, the Λ can be determined as

$$2\Lambda=\frac{\int_0^{\omega_c}d\omega'\,\omega'\,\mathrm{Im}\,\Pi(\omega')e^{-\omega'/T}}{\int_0^{\omega_c}d\omega'\,\mathrm{Im}\,\Pi(\omega')e^{-\omega'/T}}. \tag{12}$$

After all of the Λ have been calculated, the decay constants can be carried out according to Eq. (10).

III. NUMERICAL RESULTS OF THE SPECTRUM AND DECAY CONSTANTS OF THE HYBRIDS

In this context, we will give the numerical results of the spectrum and decay constants of the hybrids. To proceed with the process, the mass of the b and c quark are chosen as 4.7 GeV and 1.3 GeV, respectively, the condensates are chosen as

TABLE I. Masses of heavy-light hybrids with different J^{pc} (GeV).

J^{pc}	$2\Lambda_c$	$2\Lambda_b$	$m_H(\overline{q}cg)$	$m_H(\overline{q}bg)$	m_c(full)	m_b(full)
0^{++}	4.4	4.4	3.5	6.9	4.0	6.8
0^{--}	6.8	6.8	4.7	8.1	4.5	7.7
1^{-+}	3.6	3.6	3.1	6.5	3.2	6.3
1^{+-}	3.8	3.8	3.2	6.6	3.4	6.5
0^{++}	4.2	4.2	3.4	6.8	none	none

$$\langle 0|m\overline{q}q|0\rangle = -(0.1\ \mathrm{GeV})^4, \langle 0|\overline{q}q|0\rangle = -(0.24\ \mathrm{GeV})^3,$$

$$\langle 0|\alpha_s G^2|0\rangle = 0.06\ \mathrm{GeV}^4, \tag{13}$$

$$\langle 0|gG^3|0\rangle = (0.27\ \mathrm{GeV}^2)\langle \alpha_s G^2\rangle,$$

and the scale of the running coupling is set at the Borel parameter T.

The continuum threshold is chosen as below in the calculation: $\omega_c = 5.0$ GeV for the 0^{--} and two 0^{++} hybrids from current $j_\mu(x)$ and $j(x)$, and $\omega_c = 4.5$ GeV for the 1^{+-} and 1^{-+} hybrids.

We display our results calculated in HQET and those calculated in full QCD theory in Table. I. In this table, the right two columns represent the mass of heavy-light hybrids calculated in full theory, the left represent the results obtained in HQET and the bottom of this table represents the results of 0^{++} hybrids with the gluon in TE(1^{+-}) mode.

From the table, the mass of hybrids including the b or c quark calculated in HQET is found similar to that calculated in full theory; the light freedom in hybrids is not large enough to break down the $1/M_Q$ expansion. Therefore, the calculation in HQET is suitable, which implies that the $1/M_Q$ correction to the sum rules is not large.

Though the results are similar for the 0^{++} and 0^{--} hybrids in both methods, the situation is different for the 1^{-+} and 1^{+-} hybrids. In 1^{-+} or 1^{+-} hybrids case, the sum rules in full theory do not stabilize [7], which may indicate that no resonance exists in the channels at all. The masses of these states were given under the main assumption that the contribution of the gluon condensate is less than 20% of the bare loop. In HQET, the 2Λ of 1^{-+} or 1^{+-} hybrids versus Borel variable τ varies slowly in a large region, which is shown in Fig. 2. The dotted line represents the case of the b quark hybrid and the real line represents that of the c quark hybrid in this figure. There is little difference between them, which comes from the running coupling. The sum rules for the 1^{-+} or 1^{+-} hybrids have been improved greatly in HQET, which may come from the reason [11] that the T, 2Λ, and ω_c become constant low-energy parameters in the $M_Q \to \infty$ limit in HQET, while the dependence of the parameters M^2 and s_c on the heavy quark mass is *a priori* not determined in full theory.

In the case of light quark hybrids, the mass of different 0^{++} hybrids with the gluon in different modes was found to have a large difference [4]. However, the Λ for the two 0^{++} heavy-light hybrids with the gluon in different modes is found similar in the $M_Q \to \infty$ limit. The mass of the heavy-light hybrids in HQET is represented approximately

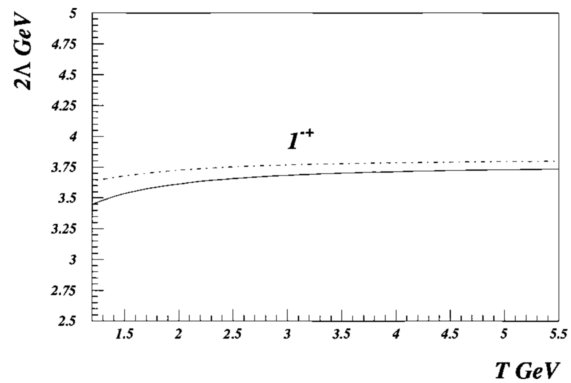

FIG. 2. 2Λ of the 1^{-+} heavy-light hybrids versus Borel variable T.

$$m \approx M_Q + \Lambda + O(1/M_Q), \tag{14}$$

from the calculation above, the mass splitting of the 0^{++} heavy-light hybrids is found not large in HQET. The mass of the 0^{++} hybrid with gluon in the TM(1^{--}) mode is about 6.9 GeV or 3.5 GeV corresponding to the b or c quark hybrid, respectively, and the mass of the 0^{++} hybrid with gluon in the TE(1^{+-}) mode is about 6.8 GeV or 3.4 GeV, respectively.

When the radiative effects are taken into account, the effective current would receive renormalization improvement and the heavy quark expansion for the full current is necessary. However, in our derivation, neither the radiative effects nor the $1/M_Q$ correction is taken into account.

The decay constants of the hybrids defined above can be obtained through formula (10); they are all collected in Table II. The result in this table shows that the decay constants of the 0^{++} hybrid with gluon in the TE(1^{+-}) mode are larger than those with gluon in the TM(1^{--}) mode.

IV. STRONG COUPLINGS AND DECAY WIDTHS OF HEAVY-LIGHT HYBRIDS

In Ref. [10], we have calculated the decay widths of process

$$H_b(0^{++})(k) \to B(0^{-+})(k-q) + \pi^{\pm}(q), \tag{15}$$

$$H_b(1^{-+})(k) \to B(0^{-+})(k-q) + \pi^{\pm}(q), \tag{16}$$

where the gluons are in the TM(1^{--}) mode and TE(1^{+-}) mode in the 0^{++} and 1^{-+} hybrids, respectively, and the two-point correlation function resulted from current $g\overline{q}\gamma_\alpha G^a_{\alpha\mu} T^a h_v(x)$. The electric charges of the mesons ex-

TABLE II. Decay constants of heavy-light hybrids (GeV$^{7/2}$).

Hybrid	F_{H^+}(TM)	F_{H^-}	F'_{H^+}(TE)
c quark	0.31	0.28	0.97
b quark	0.33	0.29	1.01

cept for pion have not been written out explicitly. The cases of $H_c(0^{++})\rightarrow D\pi^{\pm}$ and $H_c(1^{-+})\rightarrow D\pi^{\pm}$ have also been calculated there.

In this section, we will reconsider the same processes in HQET first. The decay widths of these processes are usually calculated through the ordinary three-point vertex function or QCD light-cone sum rules. However, in order to avoid the ambiguity of the three-point function resulting from the double Borel transformation and the infrared divergence in the soft pion approximation, we use the following two-point correlator between pion and vacuum:

$$A_{\nu}(\omega',\omega,v)=i\int dxe^{ikx}\langle\pi^{\pm}(q)|T\{j_{1\nu}(x),j_2(0)\}|0\rangle$$
$$=A(\omega',\omega)v_{\nu}+B(\omega',\omega)(-q_{\nu}+q\cdot vv_{\nu}), \tag{17}$$

where $j_{1\nu}(x)=g\bar{q}\gamma_{\mu}G^a_{\mu\nu}T^a h_v(x)$ and $j_2(x)=\bar{h}_v\gamma_5 q(x)$. $A(\omega',\omega)$ and $B(\omega',\omega)$ are scalar functions of $\omega=2k\cdot v$ and $\omega'=2(k-q)\cdot v$, which are determined through the spectral density saturated by the mesons corresponding to the interpolated currents, respectively. The detailed OPE expression of the $A(\omega',\omega)$ and $B(\omega',\omega)$ has been given in Ref. [10].

For the 0^{++} hybrid with gluon in the $\mathrm{TE}(1^{+-})$ mode, the current $j_{1\nu}(x)$ in the correlation function above should be replaced by $j_1'(x)=g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a h_v(x)$. Then for the processes (15), we have another correlation function

$$C(\omega',\omega)=i\int d^4xe^{ikx}\langle\pi^{\pm}(q)|T\{j_1'(x),j_2(0)\}|0\rangle. \tag{18}$$

In the infinite heavy quark mass limit, the following approximate relation

$$2\Lambda-2\Lambda'\approx\omega-\omega'=2q\cdot v \tag{19}$$

is a good one [10] and will be used in this paper, where $\Lambda\sim m_H-M_Q$ and $\Lambda'\sim m_{meson}-M_Q$. Taking into account both the single pole terms and the double pole terms in the physical side, we can express $A(\omega',\omega)$, $B(\omega',\omega)$, and $C(\omega',\omega)$ as functions of the single variable ω', respectively,

$$A(\omega')=\frac{F_{H^+}f_m g_{H^+m\pi}m_{H^+}^{1/2}m_m^2}{(2\Lambda'-\omega')^2M_Q^3}+\frac{c_0}{2\Lambda'-\omega'}, \tag{20}$$

$$B(\omega')=\frac{F_{H^-}f_m g_{H^-m\pi}m_{H^-}^{1/2}m_m^2}{(2\Lambda'-\omega')^2M_Q^3}+\frac{c_1}{2\Lambda'-\omega'}, \tag{21}$$

$$C(\omega')=\frac{F'_{H^+}f_m g'_{H^+m\pi}m_{H^+}^{1/2}m_m^2}{(2\Lambda'-\omega')^2M_Q^3}+\frac{c_2}{2\Lambda'-\omega'}, \tag{22}$$

where c_0, c_1, and c_2 are some constants. F_i is the decay constant of hybrids defined above. f_m is the decay constant of the B or D meson and $g_{H^{\pm}m\pi}$ refers to strong coupling. They are defined as

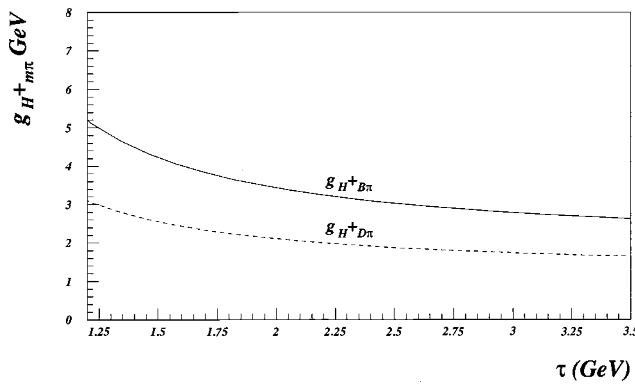

FIG. 3. Strong coupling of $H(0^{++})$ heavy-light hybrids versus Borel variable τ.

$$\langle 0|j_D|D\rangle=-if_Dm_D^2/M_c,\quad \langle 0|j_B|B\rangle=-if_Bm_B^2/M_b,$$
$$\langle\pi^{\pm}(q)D|L_I|H'(0^{++})\rangle=g'_{H^+D\pi},$$
$$\langle\pi^{\pm}(q)B|L_I|H'(0^{++})\rangle=g'_{H^+B\pi},$$
$$\langle\pi^{\pm}(q)D|L_I|H(0^{++})\rangle=g_{H^+D\pi}, \tag{23}$$
$$\langle\pi^{\pm}(q)D|L_I|H(1^{-+})\rangle=g_{H^-D\pi}\epsilon\cdot q,$$
$$\langle\pi^{\pm}(q)B|L_I|H(0^{++})\rangle=g_{H^+B\pi},$$
$$\langle\pi^{\pm}(q)B|L_I|H(1^{-+})\rangle=g_{H^-B\pi}\epsilon\cdot q,$$

where the L_I is the ordinary light quark gluon interaction vertex in the QCD Lagrangian.

The formulas (20) and (21) are a little different from those in Ref. [10] because of the different definition of the decay constants of the hybrids.

Making use of the dispersion relation and making Borel transformations on them, we will get some equations about the strong couplings. After eliminating the c_i terms with appropriate differentiation, the strong couplings are obtained:

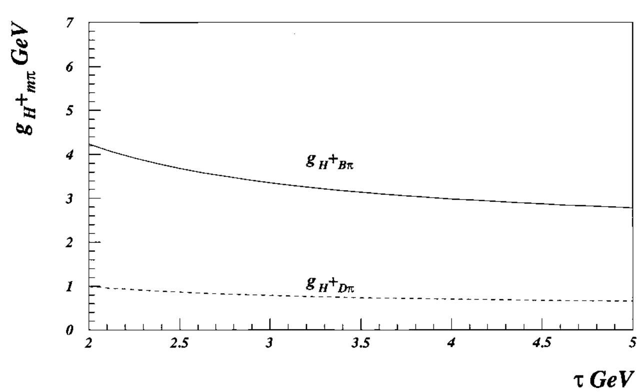

FIG. 4. Strong coupling of $H'(0^{++})$ heavy-light hybrids versus Borel variable τ.

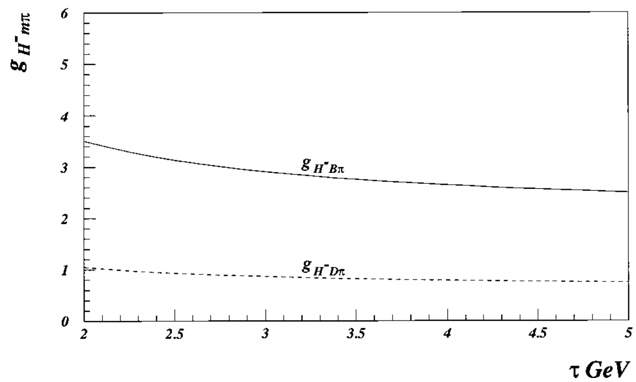

FIG. 5. Strong coupling of $H(1^{-+})$ heavy-light hybrids versus Borel variable τ.

$$g_{H^+m\pi}=\frac{M_Q^3}{F_{H^+}f_m m_{H^+}^{1/2}m_m^2}[2\Lambda' A'(\tau)+A_0]e^{2\Lambda'/\tau},$$

$$g_{H^-m\pi}=\frac{M_Q^3}{F_{H^-}f_m m_{H^-}^{1/2}m_m^2}[2\Lambda' B'(\tau)+B_0]e^{2\Lambda'/\tau}, \quad (24)$$

$$g'_{H^+m\pi}=-\frac{M_Q^3}{F'_{H^+}f_m m_{H^+}^{1/2}m_m^2}e^{2\Lambda'/\tau}[2\Lambda' C(\tau)+C_0],$$

where A_0 and B_0 have been given in Ref. [10], while $C(\tau)$ and C_0 have the form as

$$C(\tau)=8\sqrt{2}\{3[(m_H-m_m)b_2-2b_1]-(m_H-m_m)F_1/\tau\}, \quad (25)$$

$$C_0=-8\sqrt{2}(m_H-m_m)F_1, \quad (26)$$

where the parameters b_i, F_i have been calculated in Ref. [10] also.

Before going on to the numerical calculation of the strong couplings and decay widths, it is necessary to fix the parameters first. The masses of the heavy quarks and heavy mesons have been given in Ref. [12]; the decay constants of B and D mesons are chosen as in Ref. [13]. The masses and decay constants of the heavy-light hybrids have been computed above. The numerical results of the strong couplings are shown in Figs. 3–5, where the value of them is determined around the Borel variable $\tau\sim 3.0$ GeV. The results are all displayed in Table III, where the $g_{H^-m\pi}$ is dimensionless and m_H, m_m, and M_Q are the mass of the hybrid, meson, and heavy quark, respectively.

To the processes (15) and (16), the decay widths are given by the following formulas:

$$\Gamma(H(0^{++})\to m(0^{-+})+\pi)=\frac{g_{Hm\pi}^2}{8\pi}\frac{|q|}{m_H^2}=\frac{g_{Hm\pi}^2}{16\pi}\frac{m_H^2-m_m^2}{m_H^3},$$

$$\Gamma(H(1^{-+})\to m(0^{-+})+\pi)=\frac{g_{Hm\pi}^2}{24\pi}\frac{|q|^3}{m_H^2} = \frac{(m_H^2-m_m^2)^3 g_{Hm\pi}^2}{192\pi m_H^5}. \quad (27)$$

Then in the $M_Q\to\infty$ limit, the numerical results of the decay widths read

$$\Gamma(H(0^{++})\to B(0^{-+})+\pi)=86\text{ MeV},$$
$$\Gamma(H(0^{++})\to D(0^{-+})+\pi)=16\text{ MeV}, \quad (28)$$

$$\Gamma(H(1^{-+})\to B(0^{-+})+\pi)=2.2\text{ MeV},$$
$$\Gamma(H(1^{-+})\to D(0^{-+})+\pi)=1.0\text{ MeV}, \quad (29)$$

$$\Gamma(H'(0^{++})\to B(0^{-+})+\pi)=11\text{ MeV},$$
$$\Gamma(H'(0^{++})\to D(0^{-+})+\pi)=2.6\text{ MeV}. \quad (30)$$

Though the decay of hybrids follows the $S+P$ selection rule, which means that the decay of hybrids to two S-wave mesons are suppressed [14], the selection rule is not absolute. In the flux tube and constituent glue models, it can be broken by wave function and relativistic effects, and the bag model predicts that it is also possible for the excited quark to lose its angular momentum to orbital angular momentum [15]. Though the $S+P$ channels have not been calculated in this paper, the absolute numerical results for the $S+S$ channels obtained here seems to support this idea.

The decay widths for the processes to $B(D)\pi$ final states of $H(0^{++})$ with gluon in the TM(1^{--}) mode are much larger than those of $H(1^{-+})$ with gluon in the TE(1^{+-}) mode; the reason is that the final states in the later channels are in the P wave. Besides, the decay width of the $H(0^{++})\to B\pi$ obtained here is much larger than the one we obtained in Ref. [10]; the difference is from the decay constant. The decay constant of the 0^{++} hybrid we obtained there in full theory is much smaller than the one calculated above in HQET.

Though the masses of these two 0^{++} hybrids are almost the same, the strong couplings to pion of them are different, so the decay widths of these two different 0^{++} hybrids are different. The decay widths for process $H'(0^{++})\to B(D)\pi$ are smaller than those corresponding to process $H(0^{++})\to B(D)\pi$. The physical reason about the difference of the decay width between these two different 0^{++} hybrids is as

TABLE III. Some parameters input and strong couplings of hybrids (GeV).

Hybrid	M_Q	m_m	f_m	$m_H(0^{++})$	$m_H(1^{-+})$	$m'_H(0^{++})$	$g_{H^+m\pi}$	$g'_{H^+m\pi}$	$g_{H^-m\pi}$
b	4.7	5.28	0.18	6.9	6.5	6.6	8.5	3.2	2.8
c	1.3	1.87	0.19	3.5	3.1	3.2	2.0	0.8	0.8

yet unknown to us, but the difference between these two states provides us nice evidence that the decay property of these two 0^{++} hybrids with the gluon in a different mode is different.

V. CONCLUSION AND DISCUSSION

We calculate the spectrum of the 0^{++}, 0^{--}, 1^{-+}, and 1^{+-} heavy-light hybrids with different currents in HQET, the results from current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a h_v(x)$ and $g\bar{q}\gamma_\alpha\gamma_5 G^a_{\alpha\mu}T^a h_v(x)$ are compatible to those in full QCD theory. The calculation shows that the light freedom in heavy-light hybrids is not heavy enough to break down the $1/M_Q$ expansion and it is suitable to apply HQET to heavy-light hybrid systems.

The sum rules for the masses of 1^{-+} heavy-light hybrids have no platform at all in the calculation of full theory, so the masses of them were given under some assumptions. The ambiguity of these sum rules has been improved in HQET. In the calculation, the leading order $1/M_Q$ expansion approximation is used and only the first two terms in OPE of the two-point correlation function between pion and vacuum are kept, which will bring in some errors. Besides, the deviation of the decay constants of the hybrids will bring in large deviation to the strong couplings also; it is necessary to determine these decay constants more precisely.

Since the gluon can be in different modes in hybrids, the hybrids with the same J^{pc} but with the gluon in a different mode are in fact different states. In the case of light quark hybrids, the two different 0^{++} states have different masses definitely. In the heavy-light hybrids' case, the masses of these two different states are found similar in the $M_Q\to\infty$ limit, however, the calculation shows that the decay widths for the processes of these two hybrids to $B(D)\pi$ final states are different. The decay width of $H(0^{++})\to B(D)\pi$ is found to be about 86(16) MeV, while the decay width of $H'(0^{++})\to B(D)\pi$ is only 11(2.6) MeV.

The strong couplings and decay widths for the heavy-light hybrids in the processes of $H(1^{-+})\to B(D)\pi$ have been calculated also. The large difference of the decay widths calculated here from those calculated in Ref. [10] lies on the large difference between the decay constants calculated in two different ways.

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